An estimate of Earth-emitted night-light power captured by a satellite detector

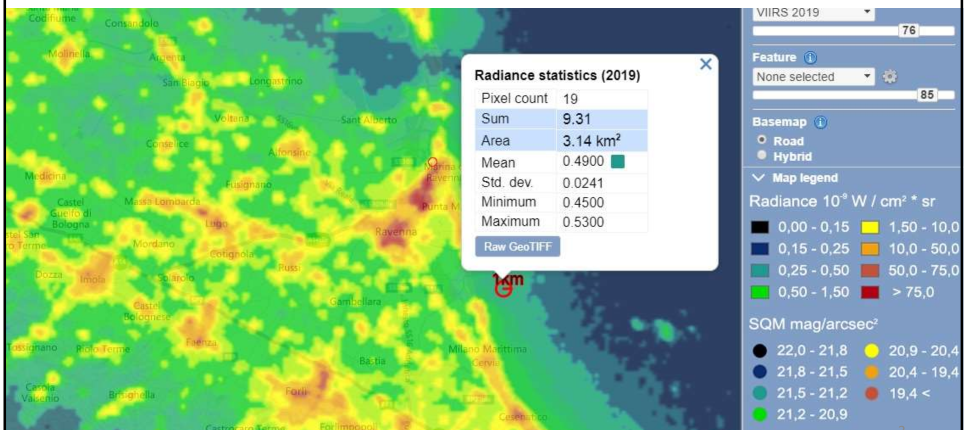
SAT010-19 Dubravko Babić
Jan 5, 2020
AOL Report 2020-01

This work has been funded by Croatian Science Foundation (HRZZ) project number IP-2018-01-2504

Contents

- Estimate the average night-light radiance (I_0 in $\mathrm{W/m^2/sr}$) from publicly available data on the internet (www.lightpollution.info). Focusing on just 500 – 900 nm range.
- Assuming altitude (h) and satellite detector diameter (d_S), compute the total power the detector will capture as a function of the field-of-view (FOV).

From the website with luight pollution data (VIIRS data) https://www.lightpollutionmap.info/#zoom=9&lat=5533081&lon=1356725&layers=B0TFFFFFFFFFTFFFF. One can determine the radiance in the 500-900nm wavelength range data. Using the built-in function on this website, I estimate the average radiance emitted is around I_0 = 10 uW/m^2/sr.



Compute the power at the satellite

- From VIIRS data, on the average, the light intensity (radiance) during the night is I_0 = 10 uW/m^2/sr.
- The total power per m^2 of Earth is given by,

$$I(\theta) = I_0 \cos\theta \quad d\Omega = \sin\theta \cdot d\theta d\phi$$

$$I_{total} = \int_0^{2\pi} I(\theta)\cos\theta \cdot d\Omega = I_0 \int_0^{2\pi} d\phi \int_0^{\frac{\pi}{2}} \cos\theta \cdot \sin\theta \cdot d\theta = \pi I_0$$

- dA_E is the emitting surface on the Earth (FOV at the surface of Earth)
- A_S is the receiving surface on the satellite (area is defined with $A_S = d_S^2\pi/4$, where d_S is the diameter of the satellite detector.

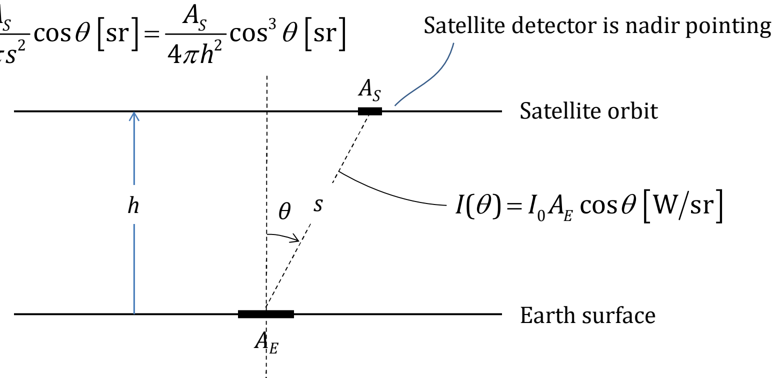
Subtleties

- Surfaces dA_E and A_S are parallel to each other
- Satellite detector acceptance angle $\Omega_S = 2\pi\cdot(1 - \cos\theta_{\max})$. The etendue of the satellite detector is $\Omega_S \cdot A_S$.
- FOV is circle with diameter $D = 2h \cdot \tan\theta_{\max}$.

5

Intensity received by A_S

$$\Omega_S(\theta) = \frac{A_S}{4\pi s^2}\cos\theta\,[\mathrm{sr}] = \frac{A_S}{4\pi h^2}\cos^3\theta\,[\mathrm{sr}]$$



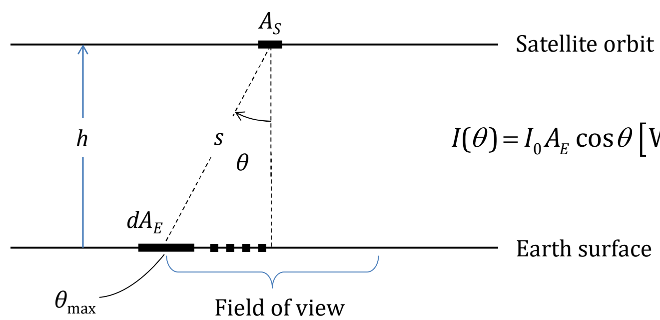
Intensity (in W) received by the satellite assuming that the receiving surface is parallel to Earth surface ($A_S \,||\, A_E$)

$$I_S(\theta) = I(\theta)\cdot\Omega_S(\theta) = \frac{I_0 A_E A_S}{4\pi h^2}\cos^4\theta\,[\mathrm{W}]$$

6

Sum A_E over the field of view

Intensity (in W) received by the satellite assuming that the receiving surface is parallel to Earth surface (A_S || A_E)



$$I(\theta) = I_0 A_E \cos\theta \left[\mathrm{W/sr}\right]$$

$$\frac{I_S(\theta)}{A_E} = \frac{I_0 A_S}{4\pi h^2}\cos^4\theta \left[\mathrm{W/m^2}\right]$$

$$P_{TOT} = \int_0^{FV}\left(\frac{I_S(\theta)}{A_E}\right) dA_E = \int_0^{FV}\left(\frac{I_0 A_S}{4\pi h^2}\cos^4\theta\right) dA_E \left[\mathrm{W}\right]$$

7

Sum A_E over the field of view

Now we sum the intensity over the entire FOV

$$dA_E = 2\pi \cdot r \cdot dr = 2\pi \cdot \left(h\frac{\sin\theta}{\cos\theta}\right)\cdot\left(\frac{hd\theta}{\cos^2\theta}\right)$$

$$= 2\pi h^2 \cdot \frac{\sin\theta}{\cos^3\theta} d\theta$$

$$P_{TOT} = \int_0^{FV}\left(\frac{I_0 A_S}{4\pi h^2}\cos^4\theta\right) dA_E = \int_0^{\theta_{max}}\left(\frac{I_0 A_S}{4\pi h^2}\cos^4\theta\right)\left(2\pi h^2 \cdot \frac{\sin\theta}{\cos^3\theta}\right) d\theta$$

$$= \frac{I_0 A_S}{2}\int_0^{\theta_{max}} \sin\theta\cos\theta d\theta$$

$$= \frac{I_0 A_S}{4}\left(1 - \cos^2\theta_{max}\right)$$

8

Sum A_E over the field of view

$$2h\tan\theta_{\max} = D_{\max}$$

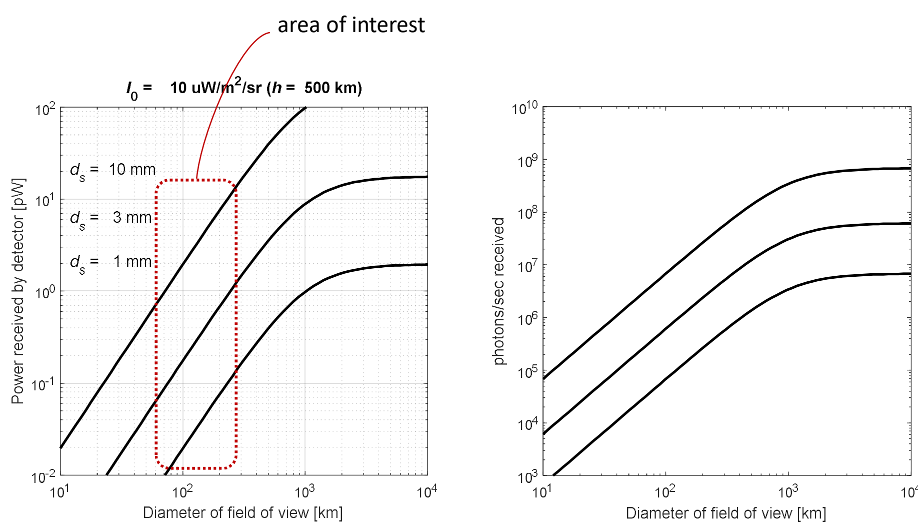
$$\cos\theta_{\max} = \frac{1}{\sqrt{1+\frac{D_{\max}^2}{4h^2}}}$$

$$P_{TOT} = \frac{I_0 A_S}{4}\left(1-\cos^2\theta_{\max}\right) = \frac{I_0 d_S^2 \pi}{16}\left(\frac{D^2}{4h^2+D^2}\right)$$

9

Results

Note that peak emission from city centers is much higher.
*Central Zagreb (120 m^2) averages to 580 $uW/m^2/sr$ (VIIRS 2019)



10

Future work

- Convert the power received into lumens to correlate to camera sensitivities.
- Estimate SNR for the TIA and detectors used